Consider the following statements. SCORE: _____ / 3 PTS

(i) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\{a_n\}$ diverges

(ii) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent

(iii) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$

Which of the statements above are true ? Circle the correct answer below.

[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

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Find all values of x for which $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$ is convergent. **You do NOT need to find the sum.** SCORE: _____ / 5 PTS

$$\left|\frac{x-2}{3}\right| < 1$$

$$-1 < \frac{x-2}{3} < 1 \quad ③$$

$$-3 < x-2 < 3$$

$$-1 < x < 5 \quad ②$$

Using mathematical induction, show that the sequence defined recursively by $a_1 = 1, \quad a_{n+1} = 3 - \frac{1}{a_n}$ SCORE: _____ / 5 PTS

is bounded. (HINT: Consider the bounds 1 and 3.) **Do NOT show that the sequence is increasing.**

$1 \le a_1 \le 3$ ①

IF $1 \le a_k \le 3$ ($\frac{1}{2}$) FOR SOME INTEGER $k \ge 1$

THEN $1 \ge \frac{1}{a_k} \ge \frac{1}{3}$ ($\frac{1}{2}$)

SO $-1 \le -\frac{1}{a_k} \le -\frac{1}{3}$ ($\frac{1}{2}$)

SO $2 \le 3 - \frac{1}{a_k} \le 2\frac{2}{3}$ ($\frac{1}{2}$)

SO $2 \le a_{k+1} \le 2\frac{2}{3}$ ①

SO $1 \le a_{k+1} \le 3$ ①

SO $1 \le a_n \le 3$ FOR ALL INTEGERS $n \ge 1$

Determine if each series below is convergent or divergent. If a series is convergent, find its sum. **SCORE: _____ / 17 PTS**

[a] $\sum_{n=1}^{\infty} \sqrt[n]{3}$ SUBTOTAL = 4 POINTS

$\underbrace{\lim_{n\to\infty}}_{1} 3^{\frac{1}{n}} = 3^0 = \underbrace{1}_{2}$

SERIES $\underbrace{\text{DIVERGES}}_{1}$

[b] $\sum_{n=1}^{\infty} \frac{3+2^n}{5^n}$ SUBTOTAL = 6 POINTS

$= \underbrace{\sum_{n=1}^{\infty} \frac{3}{5^n} + \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n}_{2}$

$= \boxed{\frac{\frac{3}{5}}{1-\frac{1}{5}}}$ 1 $+ \boxed{\frac{\frac{2}{5}}{1-\frac{2}{5}}}$ 1

$= \frac{3}{4} + \frac{2}{3}$

$= \boxed{\frac{17}{12}}$ 1

SERIES $\underbrace{\text{CONVERGES}}_{1}$ TO $\frac{17}{12}$

SUBTOTAL = 7 POINTS

[c] $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ For this series only, justify your answer **formally** (ie. by finding and using an expression for S_n)

$= \sum_{n=2}^{\infty} \underbrace{\left(\frac{1}{n-1} - \frac{1}{n+1}\right)}_{3}$

$S_n = \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots +$

$\left(\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-1}}\right) + \left(\cancel{\frac{1}{n-2}} - \frac{1}{n}\right) + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right)$

$= \frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1} = \underbrace{\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1}}_{2}$

$\underbrace{\lim_{n\to\infty} S_n}_{} = \frac{3}{2} - 0 - 0 = \underbrace{\frac{3}{2}}_{1}$

SERIES $\underbrace{\text{CONVERGES}}_{1}$ TO $\frac{3}{2}$